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... Rise Time ... for a Shallow ...

MARCH 1967

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Prepared for BALLISTIC SYSTEMS AND SPACE SYSTEMS DIVISIONS
AIR FORCE SYSTEMS COMMAND
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THE EFFECT OF RISE TIME ON CRITICAL DYNAMIC LOAD FOR A SHALLOW ARCH

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FOREWORD

This report is published by the Aerospace Corporation, El Segundo, California, under Air Force Contract No. AF 04(695)-1001.

This report, which documents research carried out for Aerodynamics and Propulsion Research Laboratory from 1 August 1966 through 1 December 1966, was submitted on 29 March 1967 to Capt. John T. Allton, SSTRT, for review and approval.

Approved

W. F. Radcliffe, Director
Engineering Sciences Subdivision
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R.A. Hartunian, Acting Director
Aerodynamics and Propulsion
Research Laboratory

Publication of this report does not constitute Air Force approval of the report's findings or conclusions. It is published only for the exchange and stimulation of ideas.

John T. Allton, Capt., USAF
Chief, Applied Mechanics Branch

ABSTRACT

The critical ramp loads required to produce snapping of a shallow sinusoidal arch are investigated. Calculated results are presented for two specific arch geometries. These results illustrate the influence of the load rise time on the level of the critical load.

CONTENTS

FIGURES

NOMENCLATURE

$a_m(\tau)$	=	generalized coordinate denoting amplitude of m^{th} component of the nondimensional displacement $\eta(\xi, \tau)$
E	=	elastic modulus of the arch material
e	=	nondimensional geometric parameter; see Eq. (2).
I	=	second moment of area of the arch cross-section
h	=	thickness of the arch cross-section
k	=	radius of gyration of the arch cross-section
L	=	span of the arch
$p(\xi, \tau)$	=	pressure load acting upon the arch
$q(\xi, \tau)$	=	$(p/EIk)(L/\pi)^4$, nondimensional pressure load
q_1^*	=	critical value of nondimensional pressure load
$\bar{q}_1$	=	$8\omega_1^2/e$, critical static buckling pressure
R, S	=	parameters in parametric resonance analysis; see Appendix
t	=	time
$w(\xi, \tau)$	=	transverse displacement of middle surface of the arch measured from the baseline of the arch
x	=	Cartesian coordinate along span of the arch
$\eta(\xi, \tau)$	=	$(1/k)\, w(\xi, \tau)$, nondimensional displacement of the arch
ξ	=	$\pi x/L$, nondimensional Cartesian coordinate
ρ_s	=	mass density of the arch material
τ	=	$t(\pi/L)^2 (EI/\rho_s h)^{1/2}$, nondimensional time

NOMENCLATURE (Continued)

τ_r - nondimensional rise time of the ramp load

τ_1 = $2\pi/\omega_1$, nondimensional period of free vibration of the fundamental symmetric mode

ω_1 = $(1+e^2/2)^{1/2}$, nondimensional frequency of free vibration of the fundamental symmetric mode

SECTION I. INTRODUCTION

The critical step pressure loads required to produce snapping of a shallow sinusoidal arch were previously determined by the author for a wide range of arch geometries[1]. These critical pressures, which were in some cases as low as 73 percent of the corresponding critical static buckling pressures, result from a particularly severe dynamic load condition since the full level of the loading is applied instantaneously. It is anticipated that the severity of the loading will be reduced in those cases wherein a finite interval of time is required for the load to develop to its maximum level. The present note describes an investigation of this effect.

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[1] Lock, M. H. "Snapping of a Shallow Sinusoidal Arch Under a Step Pressure Load," AIAA J. 4(7), 1249-1256 (July 1966).

SECTION II. ANALYSIS

We consider a shallow sinusoidal arch subject to an external pressure load. The temporal behavior of the loading is ramp-like (see Figure 1) and the spatial distribution is sinusoidal. The load is characterized by a rise time τ_r and by a maximum level q_1. The nondimensionalized pressure $q(\xi, \tau)$ is written

$$q(\xi, \tau) = q_1(\tau) \sin \xi \qquad (1)$$

The nondimensional transverse displacement $\eta(\xi, \tau)$ of the arch is approximated by the following modal expansion

$$\eta(\xi, \tau) = e \sin \xi + a_1(\tau) \sin \xi + a_2(\tau) \sin 2\xi \qquad (2)$$

where the generalized coordinates $a_1(\tau)$ and $a_2(\tau)$ denote the amplitude of the first symmetric and antisymmetric modes of free vibration; the term $e \sin \xi$ describes the initial shape of the arch. The geometric parameter e is defined by e = initial height of the arch/radius of gyration of the arch cross-section. The generalized coordinates $a_1(\tau)$, $a_2(\tau)$ are governed by a pair of coupled nonlinear ordinary differential equations. Except for the change in the time history of the load $q_1(\tau)$, these equations are identical to those treated in Reference 1.

The relief in the severity of the loading (as compared to the step pressure case) is exhibited by displaying the variation of the critical load q_1^* with the rise time parameter α where

$$\alpha = (1 + \tau_r/\tau_1)^{-1}$$

and τ_1 denotes the period of the fundamental symmetric mode[2].

[2] The rise time parameter α varies between the values of zero (static load limit $\tau_r/\tau_1 \to \infty$) and unity (step load limit $\tau_r/\tau_1 \to 0$).

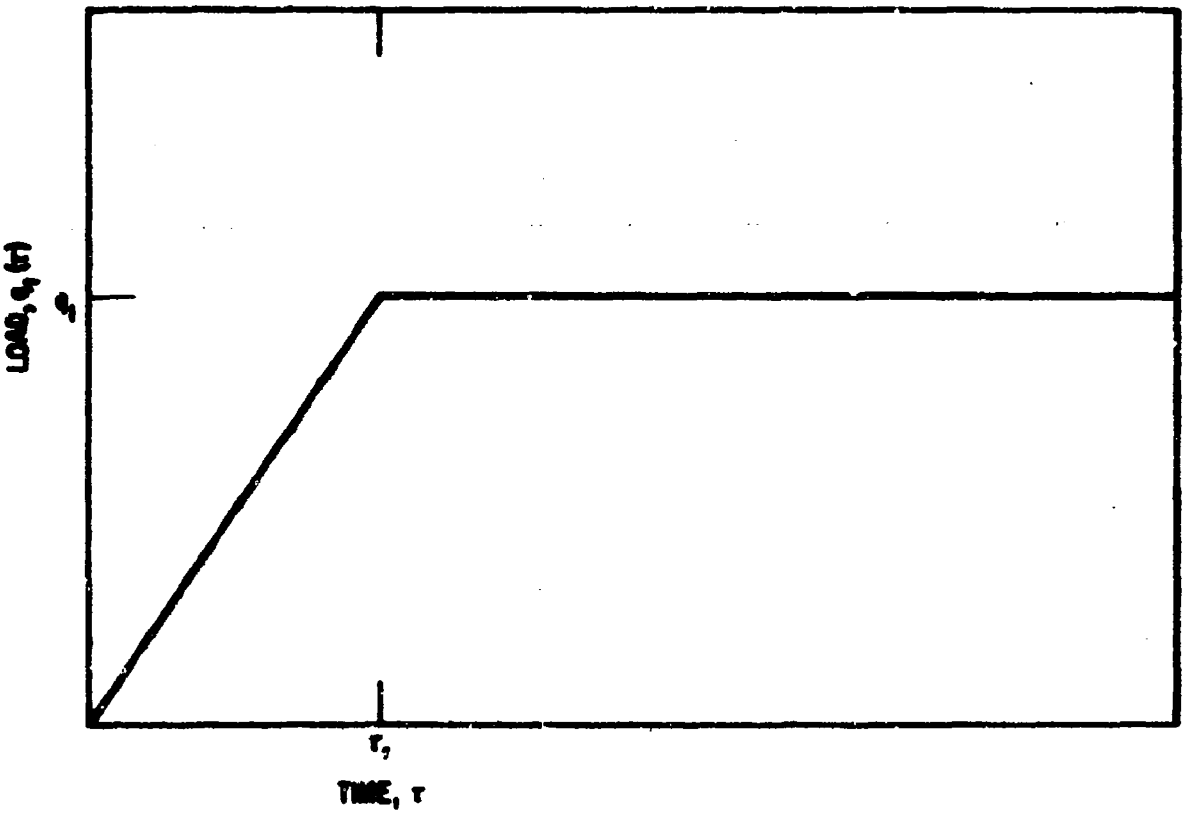

Figure 1. Schematic Diagram of Load Time History.

The critical loads q_1^* were determined by numerical integration of the nonlinear equations of motion. These loads, which are the lowest loads at which the response of the arch encompasses the corresponding snapped equilibrium state of the system (Reference 1) are revealed by a "jump" in the level of the response. Results were obtained for two specific arch geometries; namely, $e = 4$ and $e = 7$. These geometries were selected upon the basis of the results obtained in Reference 1 and are representative of the two regimes of snapping mechanism that were revealed in that study. The lower value of e corresponds to the "direct" snapping regime; the higher value corresponds to the "indirect" snapping regime.

The calculated results are shown in Figure 2 in terms of the critical load ratios $q_1^*/\bar{q}_1$, where $\bar{q}_1$ denotes the corresponding static buckling load. These ratios are plotted against the rise time parameter α. The results obtained for the two geometries are seen to be markedly different. The critical load ratio for the case $e = 4$ increases steadily as α is decreased until it reaches a maximum value of unity when α becomes zero. Thus, for $e = 4$, the level of the critical ramp load tends monotonically to the corresponding static buckling load as the rise time ratio τ_r/τ_1 is increased. In the case of $e = 7$, the variation of the critical load ratio with α is marked by the appearance of a "jump" in the value of the load ratio for $\alpha \approx 0.57$ (i.e., for $\tau_r/\tau_1 \approx 0.75$) and by the appearance of load ratios in excess of unity for $\alpha < 0.57$.

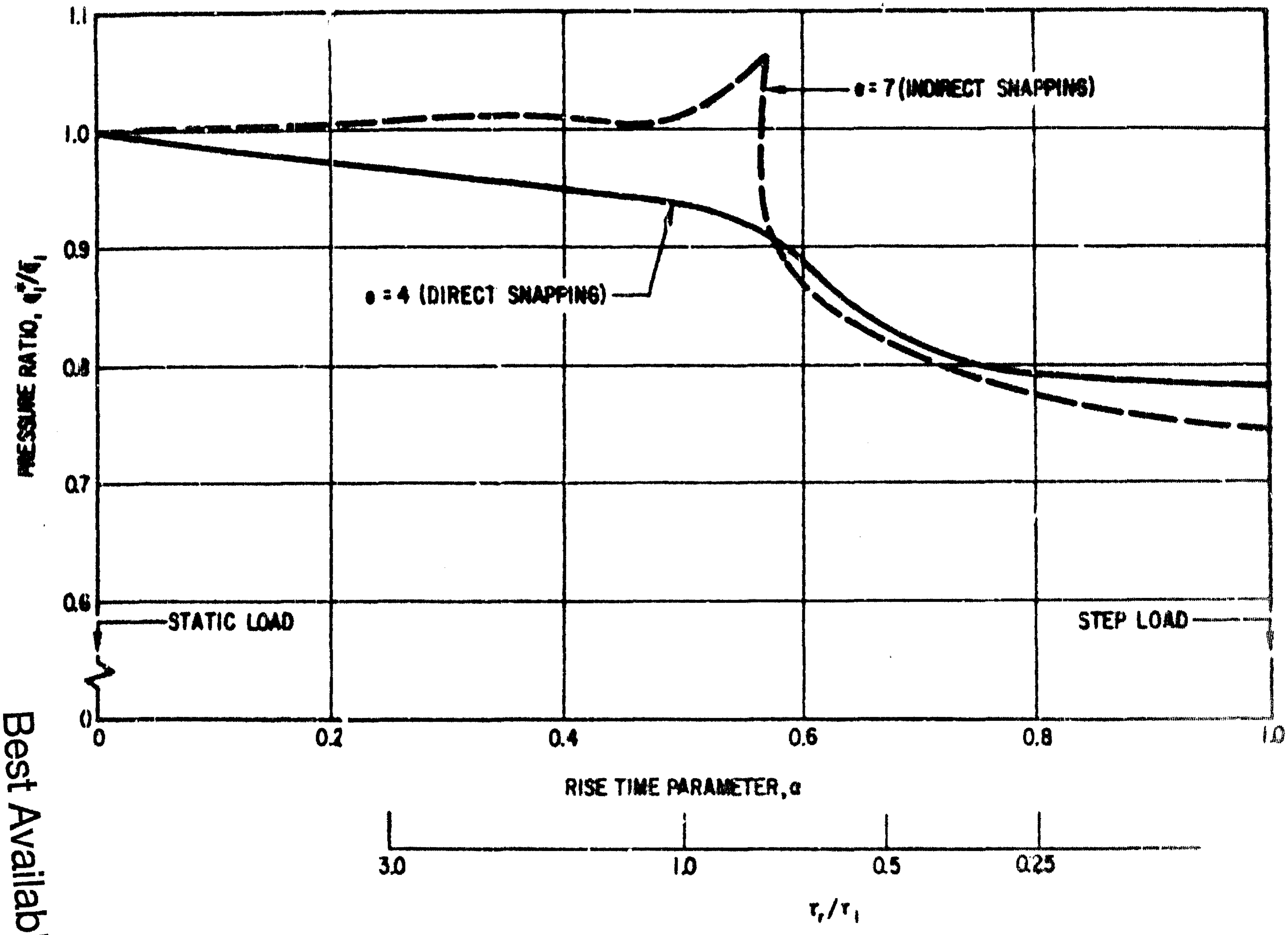

Figure 2. Variation of Critical Load Ratio With Rise Time.

SECTION III. DISCUSSION

The different character of the critical load results may be attributed to the different mechanisms of snapping that are operative in the two cases. As noted previously the critical loads for $e = 4$ are controlled by the "direct" snapping mechanism. The arch response is primarily in the symmetric mode (Figure 3) and snapping occurs when the amplitude of the generalized coordinate $a_1(\tau)$ coincides with an unstable equilibrium state (Figure 4). For $e = 7$, the occurrence of snapping is controlled by a complicated interaction between the symmetric and antisymmetric modes of the system. A more detailed discussion of the two mechanisms of snapping will be found in Reference 1. Briefly, the antisymmetric mode is parametrically excited by the initial response in the symmetric mode. If this excitation is sufficiently strong the interaction from the antisymmetric mode back to symmetric mode will precipitate snapping. This mechanism of snapping gives rise to the type of supercritical response shown in Figure 5. The presence of a "jump" in the critical load ratios and the appearance of load ratios in excess of unity may be associated with the role that parametric resonance plays in this interaction. A simple analysis to illustrate the role of parametric resonance is presented in the Appendix.

In conclusion, the presented results give an indication of the relief in critical load level that is introduced by a finite load rise time. The calculations show that the critical load levels required to produce snapping are increased from the corresponding step load values for geometries where dynamic weakening occurs (i.e., critical step loads < critical static loads). The increase is gradual if the snapping process is controlled by the "direct" snapping mechanism; the increase is more rapid if the "indirect" snapping mechanism is operative. In the latter case the dynamic weakening effects have disappeared for $\tau_r/\tau_1 < 0.75$.

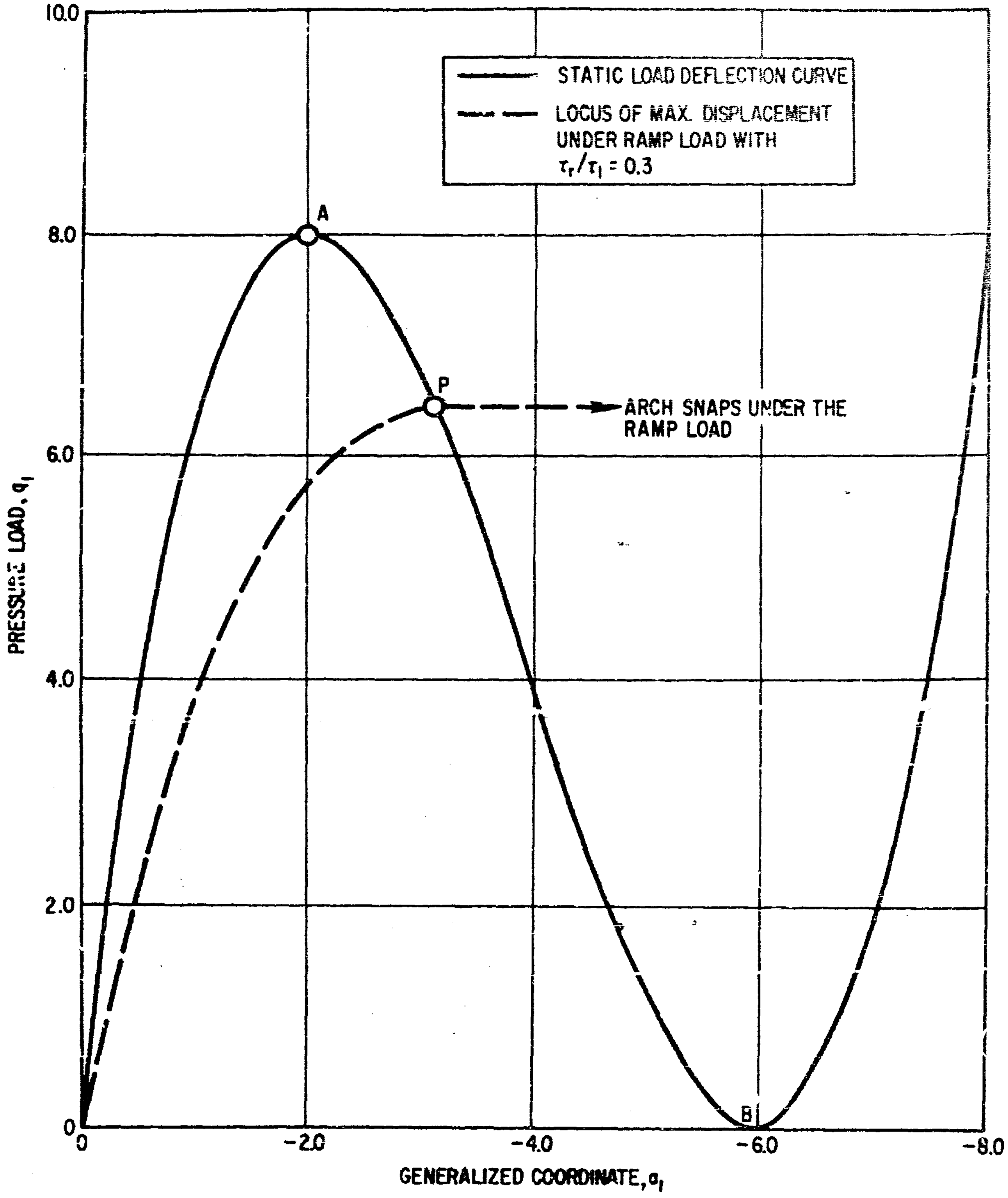

Figure 4. Load Deflection Curves for the Generalized Coordinate a_1: e = 4.

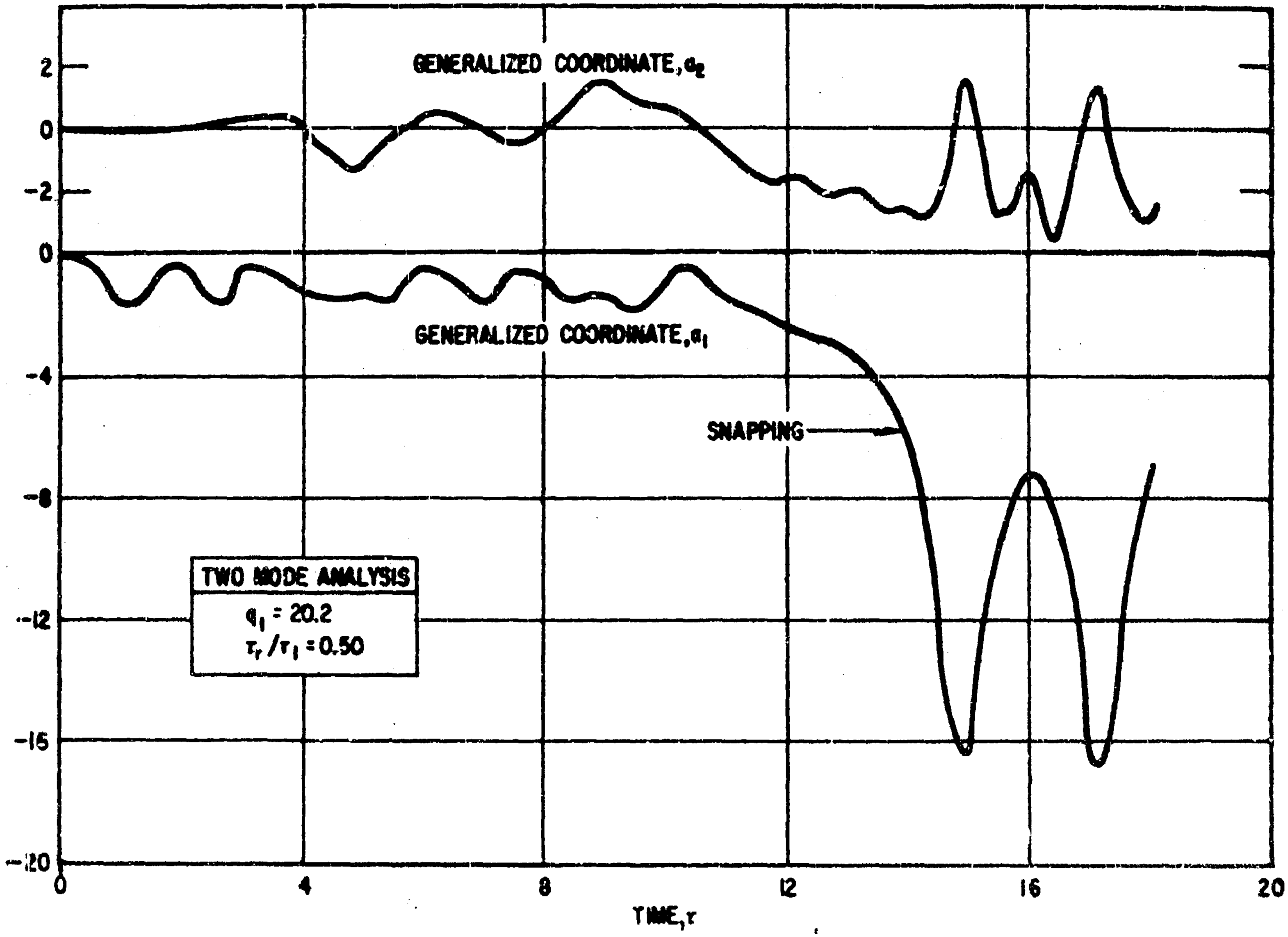

Figure 5. Response of Arch to Supercritical Ramp Load: Zero Damping and e = 7.

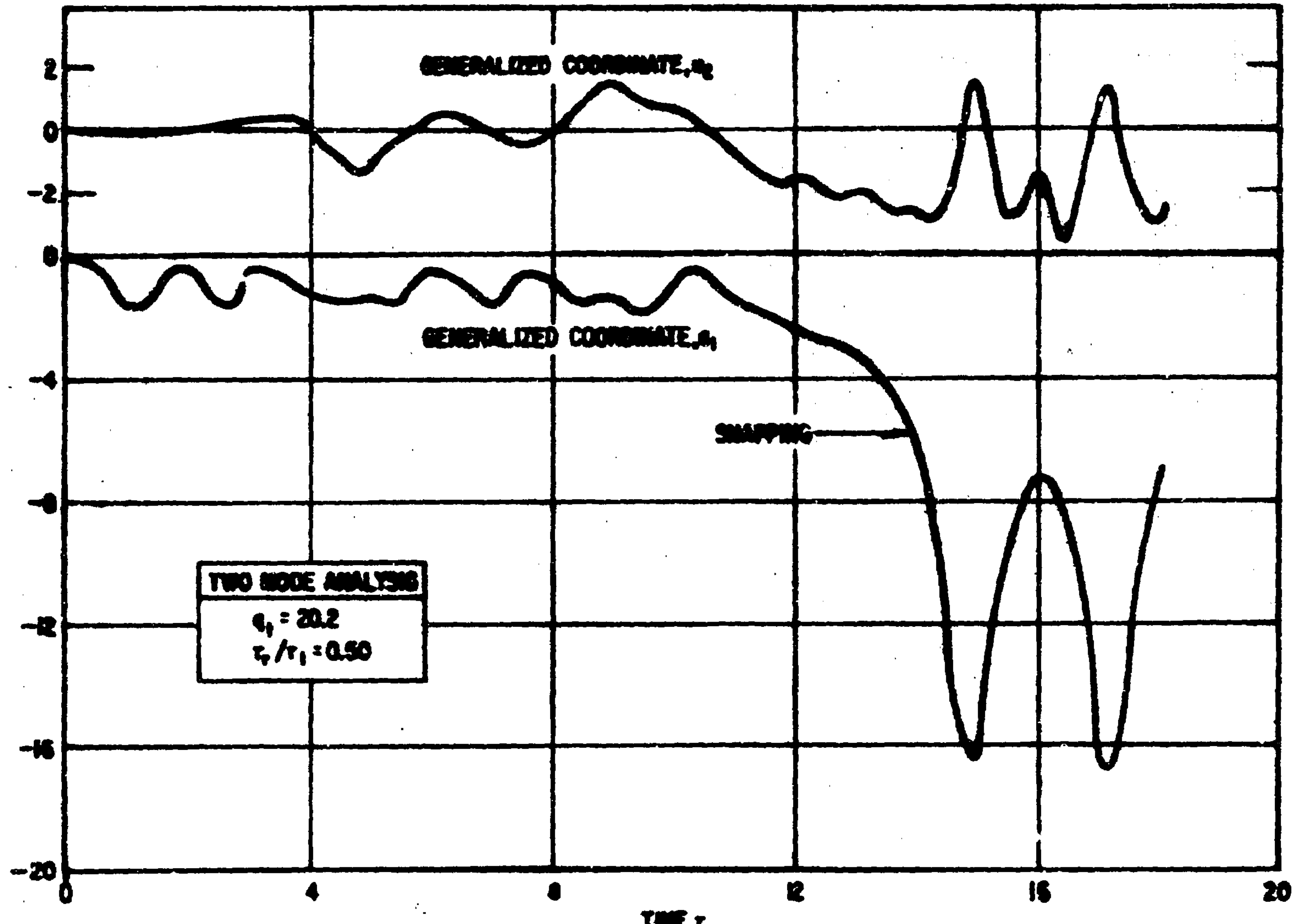

Figure 5. Response of Arch to Supercritical Ramp Load: Zero Damping and e = 7.

APPENDIX

PARAMETRIC RESONANCE ANALYSIS

The role of parametric resonance in the snapping phenomenon may be illustrated by the following analysis.

If the interaction of the antisymmetric mode response is neglected, an approximate solution for the generalized coordinate $a_1(\tau)$ is

$$a_1 \approx \frac{-q_1}{\omega_1^2}\left[1 - \delta \cos(\omega\tau + \phi)\right] \tag{3}$$

where

$$\omega_1^2 = \left(1 + \frac{e^2}{2}\right) \tag{4}$$

$$\delta = \left|\sin \frac{\pi\tau_r}{\tau_1}\right| \Big/ \left(\frac{\pi\tau_r}{\tau_1}\right) \tag{5}$$

$$\tau_1 = \frac{2\pi}{\omega_1} \tag{6}$$

and where ϕ denotes a constant phase.

The frequency ω is given approximately by the relation (see Reference 1 for method employed)

$$\omega^2 \approx \omega_1^2 - 12\left(\frac{q_1}{\bar{q}_1}\right) \tag{7}$$

If the solution (3) is substituted into the linearized version of the equation governing $a_2(\tau)$ we obtain

$$\frac{d^2 a_2}{d\hat{t}^2} + (R + S \cos \hat{t})\, a_2 = 0 \tag{8}$$

where

$$\hat{t} = (\omega\tau + \phi) \tag{9}$$

$$R = \left(\frac{4}{\omega}\right)^2 \left[1 - \frac{q_1}{\bar{q}_1}\right] \tag{10}$$

$$S = 6\left(\frac{4}{\omega}\right)^2 \left(\frac{q_1}{\bar{q}_1}\right) \tag{11}$$

Eq. (8) may be recognized as the Mathieu equation; two of the zones of parametric resonance determined by this equation are shown in Figure 6 (zones A and B in the figure). Such zones define regions where the solutions of Eq. (8) exhibit an exponential growth of the form $e^{\mu\tau}$. The iso-μ lines shown in the figure connect points of equal growth rate; the boundaries of the zone corresponding to $\mu = 0$. Considering these lines as a measure of the strength of the parametric excitation, we see that the excitation is stronger in the interior regions of the zones. The figure also shows straight line loci of the values of the load ratio $q_1/\bar{q}_1$ for fixed values of τ_r/τ_1. These loci intersect the zones of parametric resonance. As $q_1/\bar{q}_1$ is increased from zero, for fixed τ_r/τ_1, the loci first intersect zone A. If the excitation in the region of the zone traversed by the loci is sufficiently strong, snapping will occur at some critical

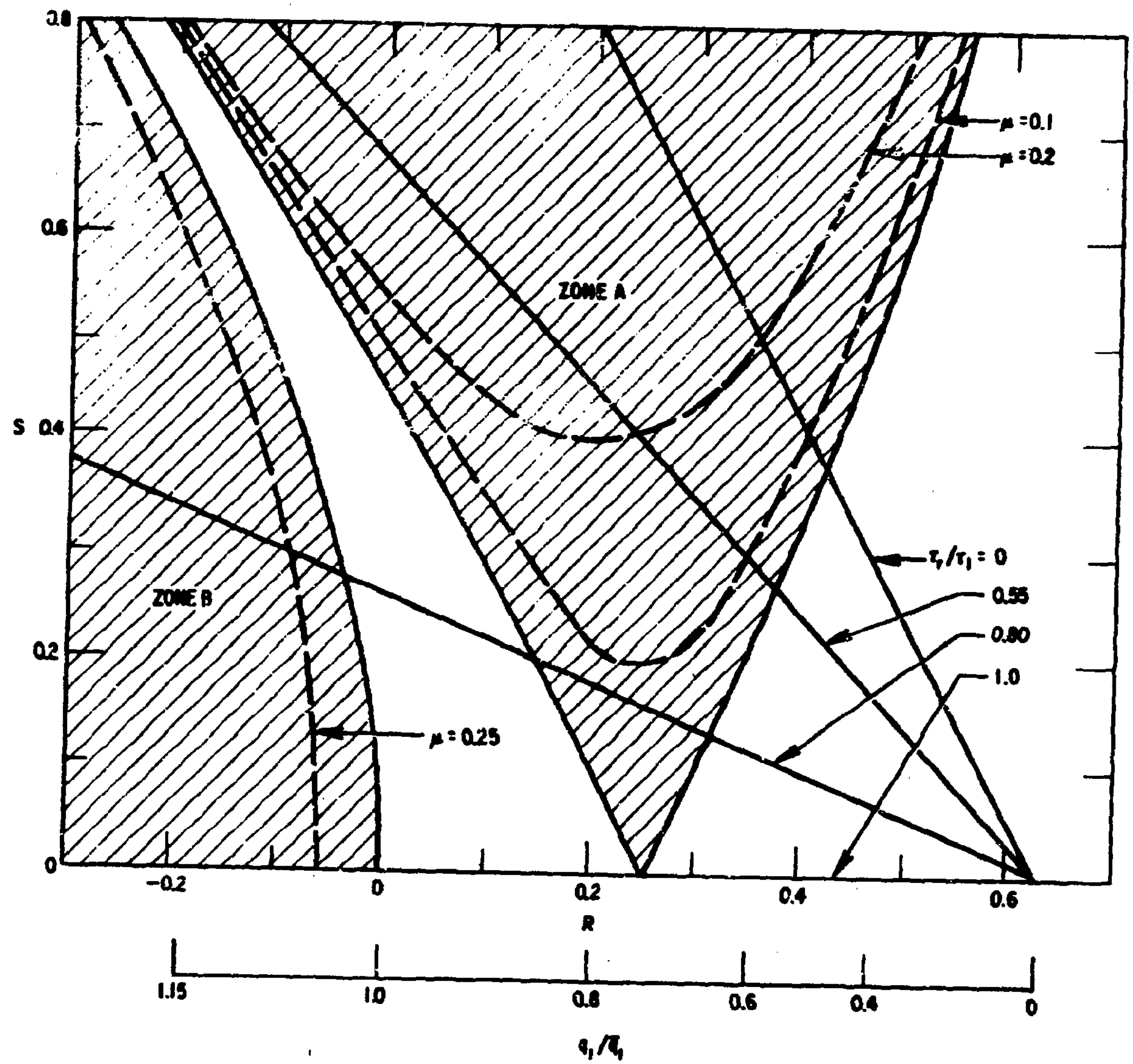

Figure 6. Zones of Parametric Resonance; Ramp Load and e = 7.

value of the load ratio. However, as τ_r/τ_1 is raised the regions of the zone traversed by the loci become increasingly weaker in excitation. Eventually the excitation in this zone becomes too weak to precipitate snapping. As a result the load ratios must be increased above unity to satisfy the conditions of parametric resonance in zone B, the snapping phenomenon now being precipitated by the excitation in this zone. Thus, it is seen that the change in the zone of parametric resonance accounts for both the jump in critical load ratio and the appearance of load ratios greater than unity (Figure 2).

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